The following examples are derived from *Homology manifold bordism* by Heather Johnston and Andrew Ranicki (Trans. Amer. Math. Soc. **352** no 11 (2000), PII: S 0002-9947(00)02630-1).

The results of Johnston [5] on homology manifolds are extended here. It is not possible to investigate transversality by geometric methods—as in [5] we employ bordism and surgery instead.

The proof of transversality is indirect, relying heavily on surgery theory—see Kirby and Siebenmann [7, III, §1], Marin [8] and Quinn [11]. We shall use the formulation in terms of topological block bundles of Rourke and Sanderson [12].

Q is a codimension q subspace by Theorem 4.9 of Rourke and Sanderson [12]. (Hughes, Taylor and Williams [4] obtained a topological regular neighborhood theorem for arbitrary submanifolds ....)

Wall [13, Chapter 11] obtained a codimension q splitting obstruction . . . .

 $\dots$  following the work of Cohen [2] on PL manifold transversality.

In this case each inverse image is automatically a PL submanifold of codimension  $\sigma$  (Cohen [2]), so there is no need to use s-cobordisms.

Quinn [10, 1.1] proved that ...

**Theorem 3.1** (The additive structure of homology manifold bordism, Johnston [5]). ...

For  $m \geq 5$  the Novikov-Wall surgery theory for topological manifolds gives an exact sequence (Wall [13, Chapter 10].

The surgery theory of topological manifolds was extended to homology manifolds in Quinn [9, 10] and Bryant, Ferry, Mio and Weinberger [1].

The 4-periodic obstruction is equivalent to an m-dimensional homology manifold, by [1].

Thus, the surgery exact sequence of [1] does not follow Wall [13] in relating homology manifold structures and normal invariants.

 $\dots$  the canonical TOP reduction ([3]) of the Spivak normal fibration of M  $\dots$ 

## **Theorem 3.2** (Johnston [5]). ...

Actually [5, (5.2)] is for  $m \ge 7$ , but we can improve to  $m \ge 6$  by a slight variation of the proof as described below.

(This type of surgery on a Poincaré space is in the tradition of Lowell Jones [6].)

## References

- J. Bryant, S. Ferry, W. Mio, and S. Weinberger, Topology of homology manifolds, Ann. of Math. 143 (1996), 435–467. MR97b:57017
- M. Cohen, Simplicial structures and transverse cellularity, Ann. of Math. 85 (1967), 218–245.
   MR35:1037
- [3] S. Ferry and E. K. Pedersen, Epsilon surgery theory I, Novikov conjectures, index theorems and rigidity, vol. 2 (Oberwolfach, 1993), 1995, pp. 167–226. MR97g:57044
- [4] B. Hughes, L. Taylor, and B. Williams, Manifold approximate fibrations are approximately bundles, Forum Math. 3 (1991), 309–325. MR92k:57040
- [5] H. Johnston, Transversality for homology manifolds, Topology 38 (1999), 673–697.
   MR99k:57048
- [6] L. Jones, Patch spaces: a geometric representation for Poincaré spaces, Ann. of Math. 97 (1973), 306–343. 102, 183–185 (1975) MR47:4269; MR52:11930.
- [7] R. Kirby and L. Siebenmann, Foundational essays on topological manifolds, smoothings, and triangulations, Ann. of Math. Study, vol. 88, Princeton University Press, 1977. MR58:31082
- [8] A. Marin, La transversalité topologique, Ann. of Math. 106 (1977), 269–293 (French). MR57:10707
- F. Quinn, Resolutions of homology manifolds, and the topological characterization of manifolds, Invent. Math. 72 (1983), 264–284. Corrigendum 85 (1986) 653. MR85b:57023, MR87g:57031
- [10] \_\_\_\_\_\_, An obstruction to the resolution of homology manifolds, Michigan Math. J. 34 (1987), 284–291. MR88i:57016
- [11] \_\_\_\_\_, Topological transversality holds in all dimensions, Bull. Amer. Math. Soc. 18 (1988), 145–148. MR89c:57016

- [12] C. P. Rourke and B. J. Sanderson, On topological neighbourhoods, Compositio Math. 22 (1970), 387–425. MR45:7720
  [13] C. T. C. Wall, Surgery on compact manifolds, 2nd ed., Academic Press, 1970.